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**Second Semester B.E./B.Tech. Degree Supplementary Examination,
June/July 2024**

Mathematics – II for CSE Stream

Time: 3 hrs.

Max. Marks: 100

- Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. VTU Formula Hand Book is permitted.
3. M : Marks , L: Bloom's level , C: Course outcomes.*

Module – 1			M	L	C
Q.1	a.	Evaluate $\int_{-1}^{+1} \int_0^z \int_{x-z}^{x+z} (x+y+z) dy dx dz$.	7	L2	CO1
	b.	Evaluate $\int_0^a \int_{\frac{x}{a}}^{\frac{\sqrt{x}}{a}} (x^2 + y^2) dy dx$ by changing the order of integration.	7	L3	CO1
	c.	Show that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.	6	L2	CO1
OR					
Q.2	a.	Evaluate $\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2 + y^2) dy dx$ by changing into polar co-ordinates.	7	L3	CO1
	b.	Find the area between the parabolas $x^2 = y$ and $y^2 = x$ using double integration.	7	L3	CO1
	c.	Using mathematical number's, write a code to find the area of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ by double integration.	6	L3	CO5
Module – 2					
Q.3	a.	Find the directional derivative of $\phi = 4xz^3 - 3x^2y^2z$ at $(2, -1, 2)$ in the direction of the vector $2\hat{i} - 3\hat{j} + 6\hat{k}$.	7	L2	CO2
	b.	If $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$ find $\text{div } \vec{F}$ and $\text{curl } \vec{F}$.	7	L2	CO2
	c.	Prove that spherical co-ordinate system is orthogonal.	6	L3	CO2
OR					
Q.4	a.	Find the angle between the normals to the surface $xy = z^2$ at the points $(4, 1, 2)$ and $(3, 3, -3)$.	7	L2	CO2
	b.	If $\vec{F} = (x + y + az)\hat{i} + (bx + 2y - z)\hat{j} + (x + cy + 2z)\hat{k}$, find a, b, c such that $\text{curl } \vec{F} = 0$.	7	L2	CO2
	c.	Using mathematical tool write a code to find the curl of $\vec{F} = x^3\hat{i} + y^3\hat{j} + z^3\hat{k}$.	6	L3	CO5
Module – 3					
Q.5	a.	Prove that the set $w = \left\{ \frac{(x, y, z)}{x - 3y + 4z = 0} \right\}$ is a subspace of $V_3(\mathbb{R})$.	7	L2	CO3

	b.	Express the matrix $M = \begin{bmatrix} 4 & 7 \\ 7 & 9 \end{bmatrix}$ as a linear combination of the matrices, $P = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, $Q = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $R = \begin{bmatrix} 1 & 1 \\ 4 & 5 \end{bmatrix}$	7	L2	CO3														
	c.	Find the basis and dimension of subspace spanned by the vectors, $\{(1,-2,3), (1,-3,4), (-1,1,-2)\}$ of $V_3(\mathbb{R})$.	6	L3	CO3														
OR																			
Q.6	a.	Find the matrix of linear transformation, $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ defined by, $T(x, y) = (2x + 3y, 4x - 5y)$ with respect to the basis, $B_1 = \{(1, 2), (2, 5)\}$ of \mathbb{R}^2 .	7	L2	CO3														
	b.	The transformation $G: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ is defined as $G(x, y, z) = (x + 2y - z, y + z, x + 2y - 2z)$. Find the basis and dimension of $\text{Im}(G)$.	7	L2	CO3														
	c.	If $f(t) = t + 2$, $g(t) = 3t - 2$, $h(t) = t^2 - 2t - 3$ and $\langle f, g \rangle = \int_0^1 f(t)g(t)dt$, find (i) $\langle f, g \rangle$ (ii) $\langle f, h \rangle$ (iii) $\ f\ $ and $\ g\ $	6	L3	CO3														
Module – 4																			
Q.7	a.	Find the real root of the equation $x^3 - 2x - 5 = 0$, correct to three decimal places using Regula-Falsi method. Carry out three iteration.	7	L2	CO4														
	b.	Using Newton's forward interpolation formula find y at $x = 5$ from the following table: <table border="1" style="margin-left: auto; margin-right: auto;"> <tbody> <tr> <td>x:</td> <td>0</td> <td>2</td> <td>4</td> <td>6</td> <td>8</td> <td>10</td> </tr> <tr> <td>y:</td> <td>0</td> <td>4</td> <td>56</td> <td>204</td> <td>496</td> <td>980</td> </tr> </tbody> </table>	x:	0	2	4	6	8	10	y:	0	4	56	204	496	980	7	L2	CO4
x:	0	2	4	6	8	10													
y:	0	4	56	204	496	980													
	c.	Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using Simpson's $\frac{3^{\text{th}}}{8}$ rule taking six equal intervals.	6	L3	CO4														
OR																			
Q.8	a.	Find the real root of the equation, $xe^x - 2 = 0$, correct to three decimal places using Newton-Raphson method. Carry out three iterations.	7	L2	CO4														
	b.	Using Lagrange's interpolation formula find $f(4)$ given, <table border="1" style="margin-left: auto; margin-right: auto;"> <tbody> <tr> <td>x :</td> <td>0</td> <td>2</td> <td>3</td> <td>6</td> </tr> <tr> <td>f(x) :</td> <td>-4</td> <td>2</td> <td>14</td> <td>158</td> </tr> </tbody> </table>	x :	0	2	3	6	f(x) :	-4	2	14	158	7	L2	CO4				
x :	0	2	3	6															
f(x) :	-4	2	14	158															
	c.	Evaluate $\int_0^1 \frac{x}{1+x^2} dx$ by trapezoidal rule considering six equal intervals.	6	L3	CO4														
Module – 5																			
Q.9	a.	Employ Taylor's series method to obtain approximate value of y at $x = 0.1$ for the differential equation, $\frac{dy}{dx} = 2y + 3e^x$, $y(0) = 0$	7	L2	CO4														
	b.	Apply Runge-Kutta method of 4 th order to find an approximate value of y at $x = 0.2$, given that $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$.	7	L2	CO4														
	c.	Apply Milne's method to find $y(1.4)$ given $\frac{dy}{dx} = x^2 + \frac{y}{2}$ and the following data : $y(1) = 2$, $y(1.1) = 2.2156$, $y(1.2) = 2.4649$, $y(1.3) = 2.7514$.	6	L3	CO5														

OR					
Q.10	a.	Using modified Euler's method, find $y(0.1)$ given $\frac{dy}{dx} = x - y^2$, $y(0) = 1$. Carry out 3 iterations.	7	L2	CO4
	b.	Using Runge-Kutta method of fourth order solve $\frac{dy}{dx} = \frac{1}{x+y}$, $y(0.4) = 1$ at $x = 0.5$.	7	L2	CO4
	c.	Using mathematical tool, write a code to solve the differential equation $\frac{dy}{dx} = x^2 + y^2$ with $y(0) = 0$, using Taylor's series method at $x = 0.1$.	6	L3	CO5
